MICHIGAN
ROSS SCHOOL OF BUSINESS

QUANTITATIVE SKILLS WORKSHOP

Self-Assessment Test

(Answers are provided at the end of the document)

1. Percentage Change

The following data are taken from the 2003 online annual report for Abbott Laboratories pharmaceutical company:

Year	Sales of Cardiology Products (millions of dollars)
2001	310
2002	473
2003	672

a. What is the percentage change in sales from 2001 to 2002? 2002 to 2003?

b. Suppose that growth in the industry is predicted to be slower from 2003 to 2004 and then to decline from 2004 to 2005. Starting from the sales figure of $672 million in 2003, sales are forecast to increase by only 6.25% in 2004 and then decrease from the 2004 level by 2% in 2005. What would be the level of sales (in millions of dollars) in 2004? In 2005? What is the total percentage change from 2003 to 2005?

2. Fractions

a. $\frac{2}{9} + \frac{1}{2} = ?$

b. Company A has, as its largest shareholder, Company B, which owns one-sixth (1/6) of the outstanding voting shares. Company A in turn owns 40% of the shares of Company C. Another one-third (1/3) of the shares of Company C are owned directly by Company B. What fraction of shares of Company C does Company B own?

3. Solving for x

Use addition, subtraction, multiplication, and division to solve for x (the solution may be a number, or it may be an expression in terms of y):

a. $2x + y = 20$

b. $y = 100 - 10x$

c. $5y = 100 + 0.1x$

d. $(y/10) = (2/x)$

4. Exponents

Simplify the following expressions for x:

a. $(x^2)(x^4)$

b. $(x^6)/(x^2)$

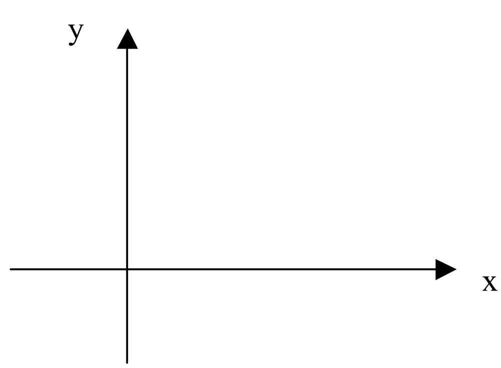
c. $(x^6)^2$

d. x^{-6}

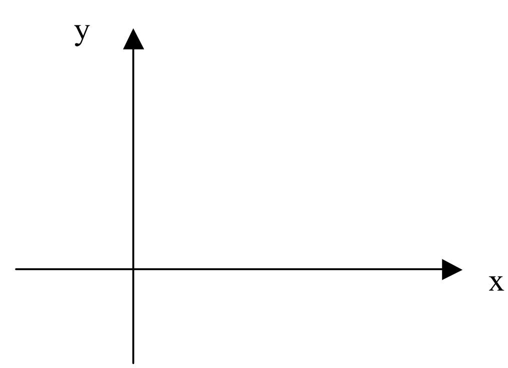
5. Equation of a Line

a. Sketch the following lines. Label the x-intercept and the y-intercept, and identify the slope of the line:

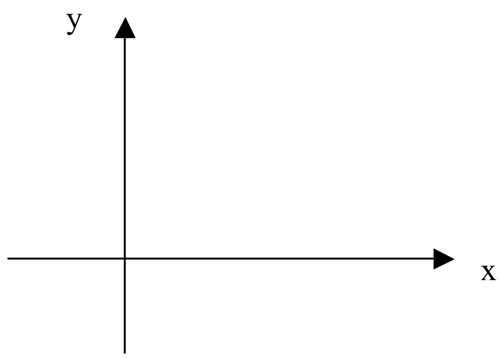
(i) $y = 10x + 500$



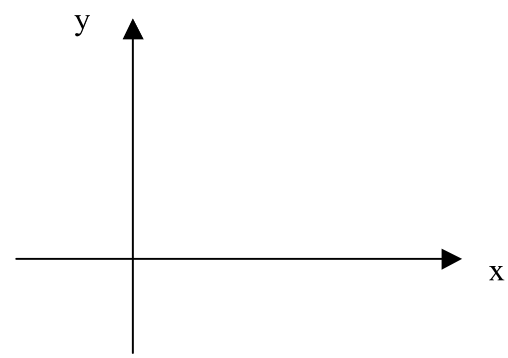
(ii) $y = 100 - 10x$



(iii) $y = 10$



(iv) $y = 2x$



b. When the price for a box of IBM-compatible diskettes was \$1.00, there were 22,000 boxes sold per month in a particular city. When the price increased to \$1.50, the number of boxes sold fell to 21,000; and when the price increased to \$2.00, only 20,000 boxes were sold. Find the equation of the line describing this demand behavior. Express the line as $Q_D = mP + b$, where Q_D is the quantity of diskettes demanded per month (in 1,000s of boxes) and P is the price per box of diskettes in dollars. Interpret, *m*, the slope of the line.

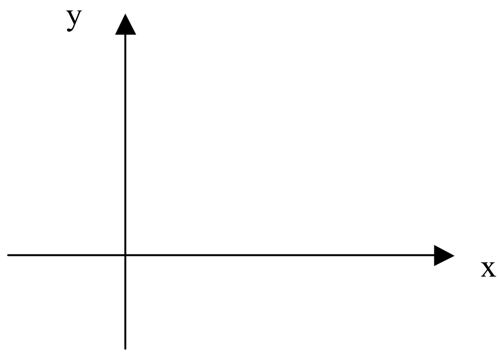
6. <u>Intersecting Lines: Finding the Solution</u>

Demand for diskettes is given by $Q_D = -2P + 24$ and supply of diskettes is given by $Q_S = 6P$, where Q_D is the quantity of diskettes demanded per month (in 1,000s of boxes), Q_S is the quantity of diskettes supplied by the industry per month (in 1,000s of boxes), and P is the price per box of diskettes in dollars. Using these lines for demand and supply, calculate the value of P for which $Q_S = Q_D$. This is called the market-clearing price or equilibrium price. Calculate the market-clearing quantity that will be sold at this price. Sketch the demand and supply lines, using the axes given below (noting that P is on the vertical axis and Q is on the horizontal axis). Label the market-clearing price and quantity on the diagram.

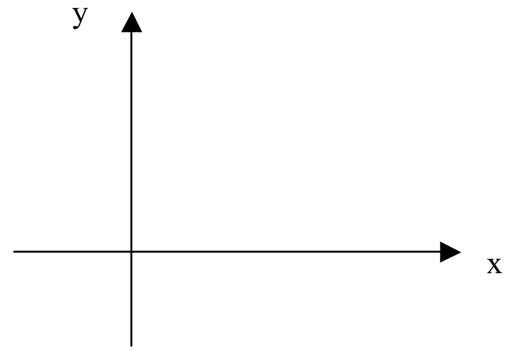
7. <u>Common Mathematical Functions</u>

Provide a rough sketch of each of the following functions for $x \geq 0$.

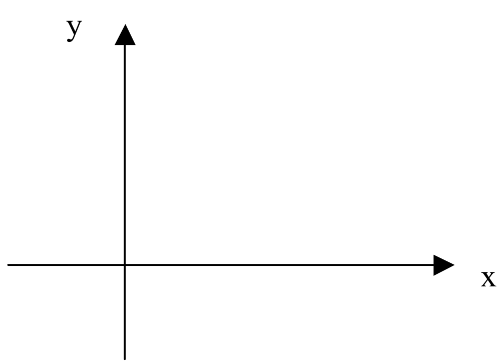
(i) $y = \sqrt{x}$ (square root)



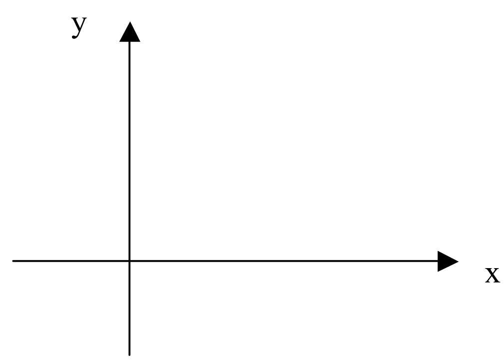
(ii) $y = 1/x$ (inverse)



(iii) $y = e^x$ (exponential)



(iv) $y = e^{-x}$ (negative exponential)



8. Sketching a Curve

Total revenue is given by the equation $R = 500Q - 5Q^2$, where R is total revenue in dollars per year and Q is quantity sold per year. Graph this function with R on the vertical axis and Q on the horizontal axis, but only for values of Q between 0 and 100.

9. Time Value of Money

Suppose you are making plans for your daughter's undergraduate education. She has just completed third grade and is therefore nine years away from enrolling as a first year college student. To date you have made no advance provision for tuition costs and wish to begin investing on her behalf. You make several inquiries to likely schools and determine that the current cost averages \$25,000 per year for tuition and fees. You estimate that these costs will grow at a rate of approximately 8% per year and that you can invest money at an after-tax rate of approximately 10% per annum.

a. Project the amount of money you will require to pay for your daughter's first year of college at the time she enrolls.

b. What sum should you invest today to accumulate the required total?

10. <u>Calculus</u>

a. Which of the following statements is *not* an accurate description of a derivative?

   (i) A derivative measures the response of y to a small change in x.

   (ii) A derivative measures the instantaneous rate of change along a function.

   (iii) A derivative measures the average rate of change along a function.

   (iv) A derivative measures the slope of the tangent line at a point on a curve.

b. Find the first derivative of y with respect to x for the following functions:

   (i) $y = 3 + 20x$ (ii) $y = 2x^4$

   (iii) $y = x^{1/2}$ (iv) $y = 9/x$

c. Every time a bottling run is begun at Chateau Ann Arbor, the winery incurs a fixed cost of \$3,200. In addition, there is also a \$2 per case cost and a \$0.2x^2 storage cost, when x cases are filled. Thus, the *total cost* of a bottling run is $TC = 3{,}200 + 2x + 0.02x^2$. If average cost is defined as the total cost divided by the number of cases filled (i.e., $AC = TC/x$), for what number of cases, x, is average cost minimized? Explain why a minimum (or maximum) can be found by setting the derivative of a function equal to zero.

RECOMMENDED SOLUTIONS

1. a. 2001 to 2002 Percentage Change = (473 – 310)/310 = 0.53 (rounding) or a 53% increase

   2002 to 2003 Percentage Change = (672 – 473)/473 = 0.42, or a 42% increase

   b. 672(0.0625) = 42. So, 2004 sales would be 672 + 42 = 714. Sales then fall by 2%, or 714(0.02) = 14.28. So, 2005 sales would be 714 – 14.28 = 699.72. The total percentage change over the two years is (699.72 – 672)/672 = 0.04 or 4% (rounding).

2. a. The lowest common denominator is 2 x 9 = 18. Multiply the top and bottom of the first fraction by 2, multiply the top and bottom of the second fraction by 9, and add:

$$\frac{2}{9} + \frac{1}{2} = \frac{2 \times 2}{2 \times 9} + \frac{9 \times 1}{9 \times 2} = \frac{4}{18} + \frac{9}{18} = \frac{4+9}{18} = \frac{13}{18}$$

   b. The fraction directly owned = 1/3. To calculate the fraction indirectly owned, you first need to note that 40 percent is 0.40 = 4/10 = 2/5. Therefore, the fraction indirectly owned = (1/6) • (2/5) = 2/30. The total fraction owned = 1/3 + 2/30 = 10/30 + 2/30 = 12/30 = 2/5 = 40%.

3. a. 2x = 20 – y ⇒ x = 20/2 – y/2 ⇒ x = 10 – 0.5y

   b. 10x = 100 - y ⇒ x = 100/10 – y/10 ⇒ x = 10 – 0.1y

   c. 0.1x = –100 + 5y ⇒ x = –100/0.1 + 5y/0.1 ⇒ x = –1,000 + 50y

   d. x(y/10) = 2 ⇒ x = 2(10/y) ⇒ x = 20/y

4. a. Add the exponents: $(x^2)(x^4) = x^{(2+4)} = x^6$

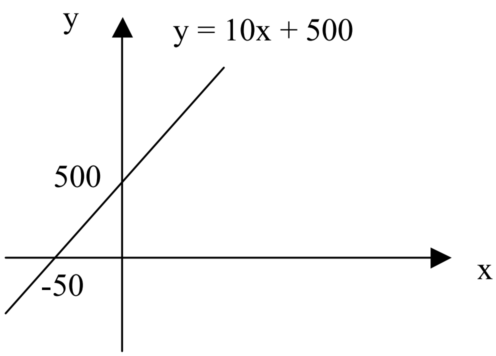
   b. Subtract the exponents: $(x^6)/(x^2) = x^{(6-2)} = x^4$

   c. Multiply the exponents: $(x^6)^2 = x^{12}$

   d. Put x in the denominator and take the absolute value of the exponent: $x^{-6} = 1/x^6$

5. a. Note that these graphs are not drawn to scale.

(i)

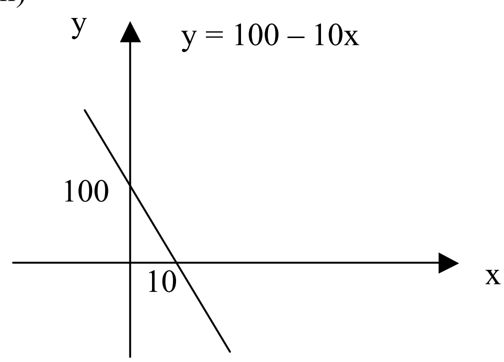


y-intercept: x = 0 ⇒ y = 500

x-intercept: y = 0 ⇒ x = -50

slope = rise/run = Δy/Δx = 10

(ii)

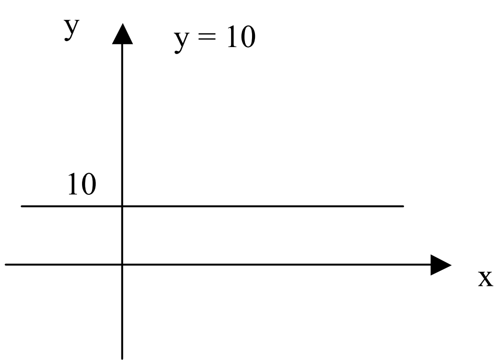


y-intercept: x = 0 ⇒ y = 100

x-intercept: y = 0 ⇒ x = 10

slope = rise/run = Δy/Δx = -10

(iii)

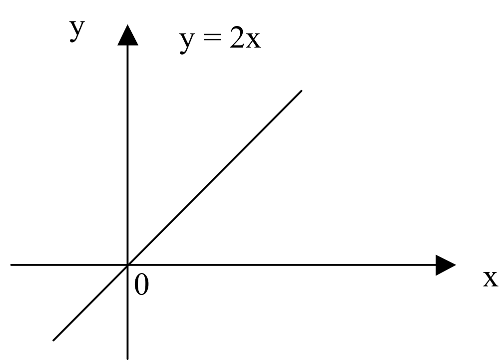


y-intercept: y = 10

x-intercept: does not exist

slope = rise/run = Δy/Δx = 0/Δx = 0

(iv)



y-intercept: x = 0 ⇒ y = 0

x-intercept: y = 0 ⇒ x = 0

slope = rise/run = Δy/Δx = 2

b. Two points determine the equation of a line. The description of demand behavior given in the problem gives us three points to work with: (22, \$1.00), (21, \$1.50), and (20, \$2.00), with quantity is measured in 1,000s. The slope of the line $Q_D = mP + b$ is m, which is the change in quantity demanded between any two points on the line divided by the change in price. For example, using the first and last points, this is: $\Delta Q_D/\Delta P = (20 - 22)/(2 - 1) = -2$. The same answer can be derived using any two of the three points given.

Given the slope of –2, we can write the equation of the line as $Q_D = -2P + b$. In order to solve for the intercept, b, plug in the values of any one of the points. For example,

$$22 = -2(1) + b$$

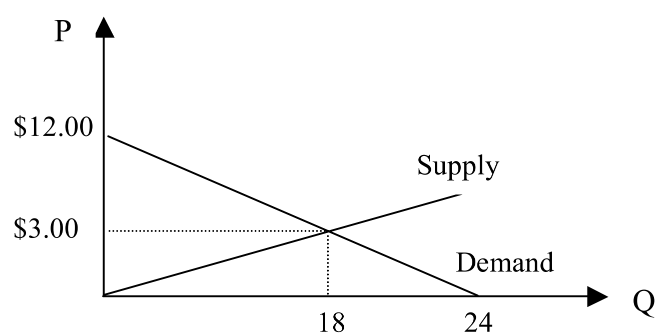
$$\Rightarrow\ b = 22 + 2 = 24$$

Therefore, the equation of the line is $Q_D = -2P + 24$.

The slope of –2 means that for every dollar increase in the price of a box of diskettes, demand drops by 2,000 boxes per month.

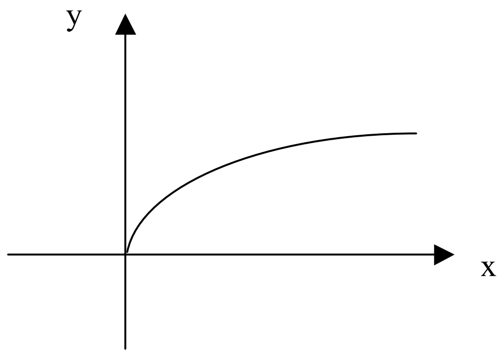
6. $Q_S = Q_D \Rightarrow 6P = -2P + 24 \Rightarrow 8P = 24 \Rightarrow P = \$3$, which is labeled on the graph below. This is the market-clearing price. Plugging P = 3 back into either the demand or supply line gives you the market-clearing quantity: $Q_D = 24 - 2(3) = 18$, or $Q_S = 6(3) = 18$.

The supply equation has a positive slope and goes through the origin. The demand line is downward sloping. The horizontal and vertical intercepts of the demand line are labeled below. Set P = 0 to find the horizontal intercept: $Q_D = 24 - 2(0) = 24$. Set $Q_D = 0$ in order to find the vertical intercept: $0 = 24 - 2P \Rightarrow 2P = 24 \Rightarrow P = 12$.

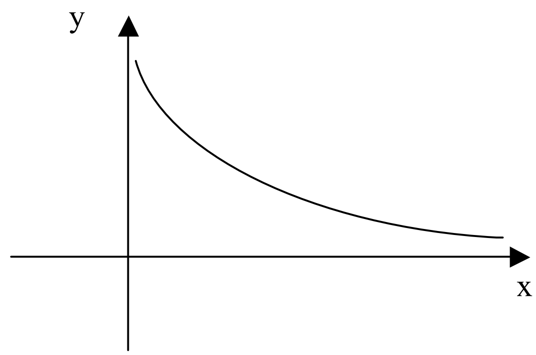


7.

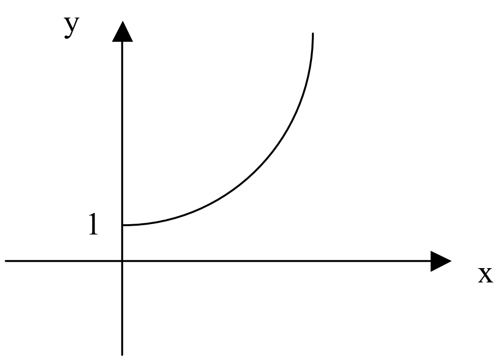
(i) $y = \sqrt{x}$ (square root)



(ii) $y = 1/x$ (inverse)

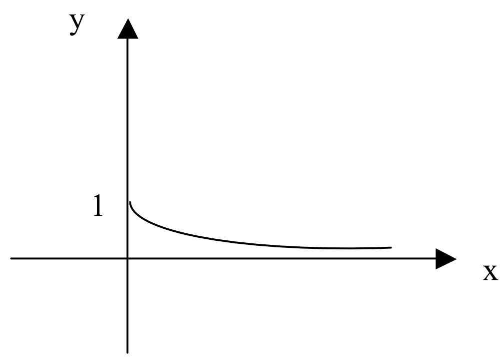


(iii) $y = e^x$ (exponential)

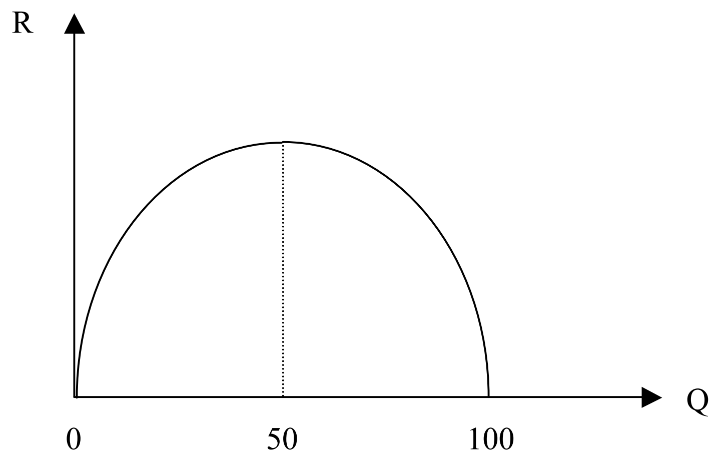


Note that $e^0 = 1$.

(iv) $y = e^{-x}$ (negative exponential)



8. The revenue function is an example of a quadratic function. The graph is shown below. Note that at both Q = 0 and Q = 100, revenue is zero. The peak of the curve is at 50.



9. Time value of money problems can be solved from first principles using the underlying exponential formulas, or with electronic calculators with time value functions. This solution illustrates both of these approaches.

   a. The amount of money needed to pay for the first year of college at the time your daughter enrolls is:

   $$25{,}000 \times (1.08)^9 = 25{,}000 \times 1.999 = \$49{,}975$$

   The factor 1.999 represents the future value of $1 projected nine years into the future at an annual growth rate of 8%. Using a financial calculator the solution can be found by entering 9 for the "number of periods"; 8 for the "interest rate"; 25,000 for the "present value"; 0 for the "annual payments" and solving for "future value." You can also calculate this with a basic calculator by multiplying 1.08 × 1.08 × … × 1.08 nine times to find $(1.08)^9 = 1.999$, and then use multiplication again to solve 25,000 × 1.999 = $49,975.

   Notice that tuition doubled over the nine-year period. This illustrates the so-called "Rule of 72's." For a number of reasonable growth rates, a sum doubles when the number of periods times the annual interest rate is approximately 72. Thus, for example, money invested at 6% will double in approximately 12 years. This is a handy rule-of-thumb to employ when approximating growth rates and future values.

b. The amount you should invest today is:

$$49{,}975 \div (1.10)^9 = 49{,}975 \times (1.10)^{-9} = 49{,}975 \times 0.4241 = \$21{,}194$$

The factor 0.4241 represents the present value of $1 discounted nine years at an annual rate of 10%. The investor is indifferent between owning $49,975 nine years in the future or $21,194 today. Using a financial calculator the solution can be found by entering 9 for the "number of periods"; 10 for the "interest rate"; 49,975 for the "future value"; 0 for the "annual payments" and solving for "present value." Or, you can use a basic calculator and multiply 1.10 × 1.10 × … × 1.10 nine times to find $(1.10)^9 = 2.358$, and then solve 49,975/2.358 = $21,194.

10. a. Statement (iii) is false. A derivative measures the slope of a tangent line to a curve at a particular point, or you can think of it as an instantaneous rate of change (or the slope for a small change in x). It is not an average rate of change.

b. First derivatives:

(i) 20 (ii) $8x^3$

(iii) $1/2x^{-1/2}$ (iv) $-9x^{-2}$

c. $AC = TC/x = (3200 + 2x + 0.02x^2)/x = 3200/x + 2 + 0.02x$, or $3200x^{-1} + 2 + 0.02x$

To find the value of x that minimizes average cost, take the derivative of AC with respect to x, and set it equal to zero:

$dAC/dx = -1(3200x^{-2}) + 0 + 0.02$

$-3200x^{-2} + 0.02 = 0 \Rightarrow 3200 = 0.02x^2 \Rightarrow x = 400$

Thus, the volume that minimizes average cost is 400 cases.

The derivative of a function tells us about the slope of that function at a point. For example, a U-shaped curve has a negative slope before it hits the minimum (the bottom of the "U") and it has a positive slope after it hits the minimum. Exactly at the minimum you could draw a horizontal line tangent to the bottom of the "U." The slope of a horizontal line is zero. Therefore, in order to find the value of x that minimizes the function, take the first derivative, set that expression equal to zero, and then solve for x. The same explanation holds for finding a maximum. For example, setting the first derivative of the revenue function in Question 7 equal to zero yields: $500 - 10Q = 0 \Rightarrow Q = 50$.

(Although the question did not ask you to do so, we should check the second derivative to make sure that x = 400 is a minimum and not a maximum. The second derivative of the AC function is $6400x^{-3}$, which is positive. Thus, x = 400 minimizes the AC function.)